

**Polynomial procedure
of avoiding multiqubit errors
arising due to qubit-qubit interaction**

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Recently proposed implementations of quantum computer suffer from unavoidable interaction between quantum bits depending upon data being written in them. Novel procedure of avoiding multiqubit errors arising due to uncontrollable qubit-qubit interaction by using additional intermediate qubits is proposed. It is shown that the scheme requires only polynomial increase in number of qubits and algorithmic steps.

Since the pioneering works of Yu. Manin [1] and R. Feynman [2] there has been a tremendous progress in quantum computation theory. P. Shor [3] and L. Grover [4] have discovered fast quantum algorithms of great practical importance. One of the most important reasons why the experimental realization of practically useful large scale quantum computer is not attained until now is attitude of quantum computers (compared to classical ones) to various types of errors. Hopefully there are known quantum error correction procedures [5–12] which help to correct errors which occurred simultaneously in single quantum bit (qubit) or in few qubits due to interaction with environment or imprecise implementation of local gates.

In 1998 J. Gea-Banacloche [13] revealed the significance of another source of errors: internal interaction in quantum computer between neighbour qubits. The interaction serves to entangle qubits if necessary, so it should be switched on every time while implementing two-qubit gate and be switched off otherwise. But the accuracy of switcher amplitude could

not be generally higher than several orders of magnitude. We should also control moments of switching off/on with such accuracy. Therefore there is weak unavoidable interaction between qubits any time. J. Gea-Banacloche [13] noted that even such a weak interaction leads to errors which could completely destroy quantum computer performance in large scale quantum registers. These errors differ from common few-qubit errors since they originate from internal qubit-qubit interaction rather than from influence of noisy environment. They conserve coherence of quantum computer but spreads over whole quantum register and make initially unentangled blocks of qubits to entangle each other. In 1999 it was pointed out by the same author [14] that common error correction methods do not solve the problem since these procedures imply that the probability of whole quantum register to be entangled due to errors during time of performing one of basic gates (I will denote this time as τ) is negligible. In the subsequent discussion it is demonstrated how to avoid these errors if we have managed to build scalable quantum computer.

As it was shown in [14] the interaction between neighbour qubits i and j frequently leads to Hamiltonian which in the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ will have a matrix like the following

$$H_{ij} = \begin{pmatrix} a^2 & 0 & 0 & 0 \\ 0 & ab & 0 & 0 \\ 0 & 0 & ab & 0 \\ 0 & 0 & 0 & b^2 \end{pmatrix}. \quad (1)$$

The inequality $a \neq b$ results in nonadditive interaction energy. It is convenient to split interaction Hamiltonian into additive H_A and nonadditive parts H_N , where

$$H_A = \begin{pmatrix} a^2 & 0 & 0 & 0 \\ 0 & \frac{a^2+b^2}{2} & 0 & 0 \\ 0 & 0 & \frac{a^2+b^2}{2} & 0 \\ 0 & 0 & 0 & b^2 \end{pmatrix}, \quad (2)$$

$$H_N = \hbar\delta\omega \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (3)$$

where

$$\delta\omega = \frac{(a-b)^2}{2\hbar}. \quad (4)$$

The dimensionless parameter δ ($\delta = \tau\delta\omega$) can be used to evaluate entanglement during one computing step. The action of H_A does not entangle qubits. Moreover, by going to an interaction picture with state $|0\rangle$ having additional energy $a^2/2$ and state $|1\rangle$ having additional energy $b^2/2$ additive part can be effectively removed. The numerical value of factor $\delta\omega$ in H_N depends on qubits being used. In some cases non-additive part can contain also off-diagonal terms, whose influence on computation performance are similar to diagonal terms [14]. The property of all such terms which is of importance for further discussion is their decrease with the increase of distance between interacting qubits R according to:

$$\delta = O(R^{-3}), \quad \text{at large } R. \quad (5)$$

This property results from dipole-dipole interaction energy scaling and it is valid if qubits interact at large distances like dipoles or weaker. It is mostly the case. For qubits with stronger entangling interaction such as proposed ones in recent paper [15] (among numerous qubit proposals I did not find other examples of such kind), where it was proposed to choose for $|0\rangle$ state the absence of electron in semiconductor quantum wire and to choose for $|1\rangle$ state the presence of electron, Eq. (5) is not true if qubits interaction is not screened by electrodes and direct implementation of technique given below does not help. But proposal of Ionicioiu *et al.* [15] can be reduced to dipole case by doubling (i.e. by polynomial increase) number of qubits. The reduction is implemented by encoding quantum information into pairs of qubits instead of single ones

$$\alpha|0\rangle + \beta|1\rangle \longmapsto \alpha|01\rangle + \beta|10\rangle$$

to provide presence of qubit and its “antiqubit” in each qubits pair.

The degradation of computer performance due to errors is characterized by probability to get right outcome after measurement — quality factor Q :

$$Q = \sum_{m \in S} |\langle m | \Psi \rangle|^2 \quad (6)$$

where S is set of problem solutions, Ψ is wavefunction of quantum register before final measurement. In abovementioned papers [13,14] it is shown that quality degrades as

$$Q \propto \exp(-\sigma^2) \quad (7)$$

where dispersion σ is given by

$$\sigma = CP\sqrt{L}\delta. \quad (8)$$

Here C is some constant whose exact value depends on algorithm being implemented and input data (usually of order of unity [13,14]), P is number of algorithm steps, L is number of qubits. The uncontrollable qubit-qubit interaction is usually weak, i.e. $\delta \ll 1$, but the numerical factor $P\sqrt{L}$ is large. For example, to factor number of 1000 decimal digits (using nowadays in RSA public key cryptography procedure) by implementing Shor algorithm [3] L should be about 10^4 and P should exceed 5×10^6 , therefore $P\sqrt{L}$ would exceed 5×10^8 . In this evaluation we do not include any error correction procedures which will also increase with both P and L in polynomial way. It should be also noted that at reasonable quantum register sizes L (at $L > 1/\delta^2$) computer strongly degrades during even single computational step. It makes impossible application of any error correction procedures to improve computing quality.

The scalability of quantum computer means that we can assemble as large uniform quantum register as we need, but can not significantly change interaction between qubits. Although obtained results can be easily generalized to two-dimensional and three-dimensional layouts of qubits, to be more concrete we restrict our consideration to one-dimensional case when all qubits are located along straight line forming one-dimensional grid with constant distance r between neighbours.

To avoid errors of interaction it is proposed to substitute logical ideal (non-interacting when it is not needed) qubits by sets of interacting qubits in the following way. Each logical qubit with number k and value a_k $|a_k\rangle$ is encoded by m real qubits

$$|a_k\rangle \mapsto |a_k \overbrace{00 \dots 0}^{m-1}\rangle. \quad (9)$$

The logical quantum register $|a_1 a_2 \dots a_L\rangle$ is encoded then by mL real qubits as following:

$$|a_1 a_2 \dots a_L\rangle \mapsto |a_1 \overbrace{00 \dots 0}^{m-1} a_2 \overbrace{00 \dots 0}^{m-1} \dots a_L \overbrace{00 \dots 0}^{m-1}\rangle. \quad (10)$$

All one-qubit logical gates V_k are performed as usual

$$V_k \mapsto V_{(k-1)m+1} \quad (11)$$

taking into account shift $k \mapsto (k-1)m+1$ of qubits numbers. Additionally in order to avoid external errors all intermediate qubits in state $|0\rangle$ should be measured during each computational step. As they are not entangled to others we can measure them without disturbing quantum coherent state of register. Nontrivial logical two-qubit gate $W_{k,k+1}$ between neighbours is performed via sequence of basic swap operators $S_{n,n+1}$

$$S_{n,n+1} |a_n a_{n+1}\rangle = |a_{n+1} a_n\rangle$$

$$S_{(k-1)m+1, km} = \overbrace{S_{km-1, km} S_{km-2, km-1} \dots S_{(k-1)m+1, (k-1)m+2}}^{m-1} \quad (12)$$

and one nontrivial two-qubit gate $U_{km, km+1}$.

$$W_{k,k+1} \mapsto S_{(k-1)m+1, km} U_{km, km+1} S_{(k-1)m+1, km}. \quad (13)$$

So it is performed via $2m-1$ basic gates. Since all qubits in state $|0\rangle$ are not in superposition state, they are prohibited from interaction entanglement [13]. By introducing such procedure interaction between neighbour qubits at distance r is effectively replaced by interaction between qubits at distance mr . Two qubits in superpositon states approach each other only during implementation two-qubit gates but in this case additional known entanglement is

not erroneous and can lead only to slight (and known in advance) change of nontrivial two-qubit gate.

Consider the influence of proposed procedure on quantum computer performance upon parameter m .

Needed space resources L' (number of qubits) are linear increased:

$$L' = mL.$$

Time resources P' (number of basic gates) are also linear increased:

$$P' \leq (2m - 1)P.$$

The equality is attained when only two-qubit gates are applied. The effective qubit-qubit interaction constant is decreased according to Eq. [5]

$$\delta' \leq \delta / m^3$$

Therefore dispersion of computation quality σ' is polynomially improved

$$\sigma' \leq \sigma m^{-3/2} \tag{14}$$

So by polynomial (upon needed dispersion change) increase of parameter m computation quality can be improved at any given qubit-qubit interaction. Finally, novel error avoiding procedure is proposed. It allows to operate with qubits interacting each other by polynomial increase of space and time resources.

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